Triple Octave Tuning

NC

June 21, 2013

Contents

Chapter 1

Introductory Remarks

Background

I was fortunate enough to be able to learn tuning at the New South Wales State Conservatorium in 1981. The course was partly funded by Yamaha, and the syllabus and teaching method were developed by Dr. Yoji Suzuki, who set up the Conservatorium School of Piano Technology adapting the methods he used in the Yamaha factory's Tuning Academy in Hamamatsu.

During the 1970s Yamaha were making up to 600 pianos a day. Each of these needed several tunings on the factory floor. In addition, the Japanese companies run service centres all round the country. The public contact the local service centre for their brand of piano, rather than an independent business as is the case in other countries.

Yamaha were responsible for about 1,800 tuning a day at the factory, and several thousand other tunings at their dealers and for the owners.

To meet these requirements, The factory trained between 200 and 250 tuners a year, each student in a separate room with a new piano. Closed circuit TV made it possible for a small team of instuctors to monitor the students' work, and the audio component also allowed for interaction.

Needless to say, Yamaha applied their trademark thoroughness to the business of teaching, and with time-and-motion studies and statistical analysis, developed the most advanced and efficient teaching method known. There has been nothing like it before, and it is unlikely there will ever be a teaching environment on the same scale again.

A unique feature of the course was the use of the Tuning Scope to mark and assess students' work. It was acknowledged that a 'Scope Tuning' was inadequate, but an aural tuning could be graphed and assessed after being measured with the 'Scope'.

This system of teaching was adapted on a smaller scale at the Sydney Conservatorium school by Dr. Suzuki who was the Head of the Yamaha Academy. He taught here for four years and during that time he developed methods suited to the Australian character and taste.

Our syllabus focused on traditional Double-Octave tuning style, and our final examination tuning was assessed as such. However during my year, while Ara Vartoukian was the Head of the School, Triple-Octave tuning was introduced as an option for anyone intending to work as a Concert Tuner.

As you know I went on to work with Ara for 25 years after that, and shared the same set of regular tunings. We only ever tuned using the Triple-Octave style. To tune quickly, accurately and efficiently requires considerable discipline. We tuned every piano, whether in the suburbs or on the Concert stage using the same style, without compromise, in order to keep our skills at their peak. Every domestic tuning was considered a rehearsal for the real thing, and by keeping the technique uniform, we maintained speed – speed is so often required in the most critical situations.

Today there are Electronic Tuning Devices flooding the profession. The market leader contains about a dozen tuning styles. I suspect that the large number of styles means that more is better. There's something for everyone there. Possibly.

Nevertheless, I find it a bit incongruous that tuners who rely on an ETD feel qualified to pick a style for themselves at whim from the dozen or so on offer.

Then again, I suppose an ETD tuner uses the same technique and muscle memory no matter what style they choose. Look at the dial and stop moving the note when the machine is happy.

It's a far cry from the discipline we worked under, constantly honing our Triple-Octave skills in preparation for the demands of the Concert Stage.

Notation Conventions

Before I launch into a discussion of triple-octave tuning, we need to get on the same page with a couple of things.

Are you comfortable with the Scientific note naming convention, where middle C is 'C4' as compared with the piano technicians' keystick notation of C40 for the same note?

I love the keystick notation[1]. It's stamped on every key. It's very handy for all sorts of reasons. With calculators and spreadsheets you can get the frequency of any note by entering the keystick number (a.k.a the key index), often denoted by 'm', into a formula:

$$frequency(m) = 2^{\frac{m-49}{12}} * 440 \tag{1.1}$$

which you can enter into your calculator as

2 ^ ((m - 49)/12) * 440

This is the notation we used at the Con, and the Yamaha course notes use it as well. The American Piano Tuners Journal *used* to use it too.

There have been several other notation systems in use. The only other one relevant to us today, is the Scientific Notation. The good old Yamaha PT100 scope which was used to mark our work, was developed by Yamaha's audio branch, and it has a modified version of Scientific Notation, in which the octave marker changes on the A's rather than the C's. That was handy on an old-fashioned scope. You had one dial to select the note by name - 'C' for instance, and another knob to select the octave by number. That's how the electronics worked.

Our work was checked using this notation, but we had to enter the data into a sheet and graph it, using the keystick notation.

Some people get passionate about the difference, and only want to use the keystick number. It's more realistic to accept both, and to move freely between them. We are, after all piano tuners, and should be adept at everything relevant to our craft, without imposing emotional limits on ourselves.

A couple of great advantages with the Scientific notation: firstly, it's easier to type (there's one less digit), but more importantly, and relevant to the discussion of octaves, double octaves and triple octaves, is that it's obvious that, for instance, from F3 to F5 you have an interval of two octaves, and similarly from F3 to F6 there are three octaves. It's much less obvious if you say that there are three octaves between F33 and F61.

The choice is parallel to measuring in metric or imperial. If some part of a piano is exactly 12" wide, why announce that it's 304.8mm? But if you're going to enter the size into a table-saw which only has metric calibration, by all means use the metric version of the number.

[1] Other names are 'key index', 'factory notation' or 'technician's notation'.

Converting from Key Notation to Scientific Notation

There's an easy way of going back and forth between the notations:

Middle C, key-index C40, is C4 in Scientific.
Middle B, key-index B39, is B3 in Scientific.

For much of the middle area, the 40's relate to the 4 octave, the 30's relate to the 3 octave, etc. It's not 100% accurate, as the key indexes change in groups of 10, while the Scientific notation, based on octaves, changes in groups of 12.

If you're into mental arithmetic, here's something to exercise your mind with while tuning.

Take the keystick number, 41, for instance, (which we know already is C♯).

Add 8. You have 49.

Divide by 12. You have 4, and 1 remaining.

So this note is in the 4th octave, and is one semitone above C, so it's C♯4.

Do it again with key number 49.

Add 8, you have 57.

How many 12s in 57? Answer 4 and 9 semitones remaining. A is 9 semitones above C, so key number 49 is A4.

The system works well all across the piano, but in the lowest octave you need to add 20 (8 + 12) and subtract 1 from the octave result.

So for note 1, add 20, you have 21. Divide by 12 you have 1 and 9 remaining.

Subtract 1 from the octave result, 1 - 1 = 0.

So it is in the zero'th octave, and is 9 semitones above C0, so the answer is A0.

Dollars and Cents

We also need to be on the same page about semitones and cents. I think all tuners are comfortable with the idea of cents. If a piano is about half a semitone flat, we tell the owner that it's 50% of a semitone flat ("It must be at least 10 years since it was last tuned") and prepare them to be fined for leaving the piano so long.

It's handy to do a few calculations involving cents - that way you it all makes more . . . sense.

Suppose you're writing an invoice. The tuning was $170, but this time you're going to add the GST. You instinctively know to add 17 to 170 and come up with $187. Some people don't trust themselves. They get out a calculator and go '170 + 10 % = ' and the answer '187' pops up for them.

Others might use the calculator a little differently, and enter '170 x 1.1 = ' to get the same answer. The 'x 1.' bit means multiply 170 by 1 to get ... 170, and the '.1' bit means 'and add 10%'.

Suppose you put $100 dollars away in a bank account, with an annual rate of 6%. You want know two things: how much you'll have after a year, and how long it will take to double your money.

You could fire up your calculator, enter: ' 100 + 6 % = ' and you'd see the amount, 106.0. Or you could use the second method and go: '100 x 1.06'. This time the '.06 part means 'and add 6%'.

It turns out that it would be around 12 years before your original $100 doubled. You could work it out by doing the multiplication 12 times:

100 x 1.06 x 1.06 x 1.06 x 1.06 x 1.06 x 1.06 x 1.06 x 1.06 x 1.06 x 1.06 x 1.06 x 1.06 and then you'd see your answer, 201.22, which is a tiny bit more than double.

That's a lot of work, entering the multiplication 12 times. It's much more 'powerful' if you go '100 x 1.06 *to the power of* 12. On the calculator:

100 * 1.06 ^ 12 =

and your answer, 201.22 pops up.

Now suppose you wanted to know what annual investment rate would give you *exactly* twice the amount, over twelve years. Of course, you'd be happy with a little extra, but you are curious to know the precise answer. What would it be if the rate was 5% ? So you'd do your *power* calculation again:

100 * 1.05 ^ 12 =

and find that it would be considerably less than double: 179.58. You could keep going all afternoon trying to satisfy your curiosity about the investment rate which would double your money exactly, to the last cent, over 12 years.

This is precisely the question which instrument makers were asking in the 16th century. Obviously there were no calculators. Less obvious is that the maths needed to solve the problem hadn't been developed, and when it was, it took quite a while till it ended up in the hands of these instrument makers. The guitar makers and luthiers were using an approximation, the '18/17' rule for setting the distance between frets.

If you do the calculation, $18 \div 17$ is close to our 1.06 rate: it's 1.05882353. More decimal places, but if we apply it to our twelve-year investment example,

$100 * 1.05882353 ˆ 12 = $198.56

the result is short of $200 by $1.44, so it's not quite as close as 1.06 gave us, which was too much by $1.22 [2].

So the number we are looking for is somewhere between 1.06 and 1.058, and to cut a very long afternoon short, it's 1.0594631.

If you were to multiply your original investment of $100 by 1.0594631 12 times ('1.0594631 *to the power of* 12') you get $200. Not only will this number magically *exactly* double an investment over 12 years, it will also help you calculate the distance between frets on your viola da gamba, or help you find the frequency of notes. It can also be used to help us describe how much sharper a double octave is than a simple octave on a piano.

There's a lot this number can reveal, it should be a fundamental part of every piano tuner's mental tool kit. In the Yamaha course, we had to memorize it, and use it in our theory tests which were conducted on Friday afternoons.

Before we look at some of the interesting things 1.0594631 will do for us, we should give it a name.

The Twelfth Root of Two

We saw that if you multiply $100 by 1.0594631 twelve times, you end up with $200. If you only had $1 to start with, you'd get $2. Taking the dollars out of the equation:

1 * 1.0594631 ˆ 12 = 2

or more simply

1.0594631 ˆ 12 = 2

which is like saying '1.0594631 multiplied by itself 12 times equals 2'. The name for our special number is "*The Twelfth Root of Two*". What's with the 'Root' word here?

2 x 2 = 4 2 is the 2nd root of 4
2 x 2 x 2 = 8 2 is the 3rd root of 8
2 x 2 x 2 x 2 = 16 2 is the 4th root of 16
2 x 2 x 2 x 2 x 2 = 32 2 is the 5th root of 32
2 x 2 x 2 x 2 x 2 x 2 = 64 2 is the 6th root of 64

[2] It actually works quite nicely for guitars, in that it helps compensate for the extra tension you get as you move along the fingerboard, due to the increasing height of the string as you move along the fingerboard.

2 x 2 x 2 x 2 x 2 x 2 x 2 = 128 2 is the 7th root of 128
2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 = 256 2 is the 8th root of 256
2 x 2 X 2 x 2 x 2 x 2 x 2 x 2 x 2 = 512 2 is the 9th root of 512
2 x 2 x 2 X 2 x 2 x 2 x 2 x 2 x 2 x 2 = 1024 2 is the 10th root of 1024
2 x 2 x 2 x 2 X 2 x 2 x 2 x 2 x 2 x 2 x 2 = 2048 2 is the 11th root of 2048
2 x 2 x 2 x 2 x 2 X 2 x 2 x 2 x 2 x 2 x 2 x 2 = 4096 2 is the 12th root of 4096

In the same way you can abbreviate '2 x 2' to 2^2 , there is a short way of writing the root of a number, and that is with the 'root' sign $\surd$ so the table above with all the 2's, can be written more concisely like this:

$$\begin{array}{ll} 2^2 = 4 & 2 = \sqrt{4} \\ 2^3 = 8 & 2 = \sqrt[3]{8} \\ 2^4 = 16 & 2 = \sqrt[4]{16} \\ 2^5 = 32 & 2 = \sqrt[5]{32} \\ 2^6 = 64 & 2 = \sqrt[6]{64} \\ 2^7 = 128 & 2 = \sqrt[7]{128} \\ 2^8 = 256 & 2 = \sqrt[8]{256} \\ 2^9 = 512 & 2 = \sqrt[9]{512} \\ 2^{10} = 1024 & 2 = \sqrt[10]{1024} \\ 2^{11} = 2048 & 2 = \sqrt[11]{2048} \\ 2^{12} = 4096 & 2 = \sqrt[12]{4096} \end{array}$$

So we can now write some equations involving *The Twelfth Root of Two* using this notation:

Firstly,

$$\sqrt[12]{2} = 1.0594631$$

'The Twelfth Root of Two is 1.0594631'.

and

$$1.0594631^{12} \ = \ 2$$

'*1.0594631 multiplied by itself 12 times equals 2*'.

There have been a lot of numbers in this section. No need to fully understand every bit of it; the main thing is to remember the actual number. And remember that the 2's and 12's in those equations hint that 1.0594631 has something to do with doubling frequency over an octave, in twelve stages – the semitones.

Calculating Cents

The calculations we had to do in the Yamaha course every week always involved 1.0594631. I doubt doing this actually makes your tuning improve by itself, but it helps your understanding of what you're doing, and that can't be a bad thing. Besides, getting the answers right got you marks, which helped with your assessment, and ultimately your reputation.

Here's a sample of the sort of exercise we did:

Question 101.1
The frequency of A49 is 440 cycles per second. Find the frequency of A♯50, and show the number of cycles per second difference.

Answer:
The frequency of A♯ 50 is 440 x 1.0594631 which is 466.163764. The difference between the frequencies is:

$$466.163764 - 440 = 26.163764$$

Question 101.2
There are 100¢ in a semitone. Show the number of cycles per second in a cent between A49 and A♯50.

Answer:
There are 26.163764 cycles per second in the 100¢ between A49 and A♯50. Therefore there are 26.163764 / 100 cycles in a cent between A49 and A♯50.
26.163764 / 100 = 0.26163764 .

Question 101.3
Using your answer to Q101.2 calculate the frequency of A49 when it is 4 cents sharp.

Answer:
4 x 0.26163765 = 1.04655056
The frequency of A49 when it is 4¢ sharp is 440 + 1.04644056 = 441.04644056

Question 101.4
What can you say about the commonly used frequency of 441 cycles?

Answer:
441 is very close to 441.04644056, so 441 is about 4 cents sharp.

A more accurate way of calculating Cents

If the number of cycles per second between semitones increases with each semitone, as calculated using the *Twelfth Root of Two*, it only takes a small spark of imagination to wonder whether the difference in frequency increases from cent to cent in a similar way. In the Tuning School calculations in the previous section, the complete semitone is divided into 100 equal parts, so each cent has equal value.

A finer resolution can be obtained if you divide the octave, not into 12 parts, but into 1,200 parts using the *Twelve-hundredth Root of Two* or $\sqrt[1200]{2}$. This can also be expressed as *two raised to the power of one over twelve hundred* and written as $2^{\frac{1}{1200}}$ and entered into a calculator or spreadsheet as 2 ^ (1 / 1200).

Using this finer resolution, we can work out the frequency of A49 when it is 4 cents sharp much more quickly, entering

2 ^ (4 / 1200) * 440

into the calculator, and finding the frequency to be 441.01779.

I need to add a paragraph on the inverse - finding the pitch in cents given a frequency.

Harmonics or Partials? Overtones?

These are different ways of describing much the same thing. Overtones don't really concern us in piano technology. The usual use of the word is to describe the higher tones of drums and other sound sources. Frequencies higher than the fundamental on such instruments are not as neatly structured as they are in 'harmonic' sources, like wind columns, or vibrating strings.

'Harmonics' and 'partials' refer to much the same thing. The Fundamental of a string can be thought of as the *first* harmonic, and the second harmonic is the sound achieved by partly stopping the string half-way along its length.

We know this.

The third harmonic is found at the points which divide the string into three parts, and produces a sound at the level of an octave-plus-a-fifth. The fourth divides the string into four parts.

Music theory uses the term 'harmonic' for these components of string or woodwind sounds; the term harmonic suggests the idea that these higher tones have a harmonious relationship to the fundamental tone of which they are part.

Physics names the same phenomena 'partials' - not to be different, as such, but rather the term suggests the physical make-up of the components of a vibrating string or column of air. The string can be thought to be vibrating in *parts*. So while the first partial vibrates along the whole length of the string, the second partial divides the string into 2 equal parts, the third partial divides the string into 3 equal parts, and so on.

Very much the same description as the harmonic description, *with one major difference*, and that is that Physics also includes an unheard zero-th partial as the Fundamental of the string.

How so?

The supposedly 'harmonic' second partial *should* be vibrating at twice the frequency of the fundamental, three time for the 3rd partial, four times for the 4th partial and so on. But the partials of piano strings, are slightly out of tune, they are in fact *inharmonic*. Not as grossly out of tune as the overtones of a drum-skin, but close enough to be pass as harmonious partials.

You can picture a piano string as an ensemble of at least 20 players[3]. Let's say there are 20 players for middle C. The first part(ial) actually plays the note, the 2nd part(ial) plays it an octave above, usually not as loud, but somewhat out of tune, perhaps 1 or 2, even 3 cents sharp.

The third part(ial) is usually a little quieter, and about as sharp as the second part.

Occasionally, especially in notes like G which are overworked and underpaid, the second and third parts play a little flat, to see if anyone notices. No-one ever does, although the tuner might find they have trouble tuning the notes with really worn out hammers in the temperament. Only when a technician voices their string do they agree to play like the other parts in the other strings, slightly sharp.

The fourth part should be a little quieter, but tries to stand out a bit by being even sharper. And so on. The really high parts play really quietly. They're expected to behave like the lower parts, playing somewhat sharp, but they seem to get it right more often. The frequencies of the higher partials are important to the analysis of the pitch of the note as a whole, and point out the irregularities in the behaviour of the louder lower parts.

Physics, with the aid of measuring eqipment, looks at all the parts it can identify, and averages out all the sharpness and flatness and comes up with a pitch for the string which is slightly different to the pitch of the first part(ial), which is what musicology thinks is the pitch of the string.

So you can have an ensemble called A4, whose 1st part(ial) is playing right on 440hz. All the other parts are playing a bit out, and so when they're averaged out, this perfectly tuned A4 might be found to be 1 cent flat, for example. This means that the first partial, which is playing so right, is in fact 1 cent sharp of the average, the zero-th partial. In Physics the term *Fundamental* refers to this partial, abbreviated at times as F_0.

This is in contrast to Musicology, which calls the 1st partial the Fundamental. Both sciences are right, the perspective is different.

In the well-tuned A4 example just discussed, if you played that note in isolation,

[3] Some notes don't need the full ensemble. The top notes often get away with only two players. The bass notes need a full orchestra, although quite often the bass-player him- or her-self doesn't bother actually making a sound, as far as anyone can tell. The other parts cover for them.

you would hear it as 1 cent flat, but there would be nothing to compare it with, and if you had perfect pitch, it's so close you'd still say it was right.

If you played the same note in combination, perhaps with A3 below it, you'd hear its first partial, among others, not the zero-th. The two pitches for the same note can be distinguished as *The Acoustic Pitch* (the averaged-out pitch) and the *Perceptual Pitch* for the pitch which you hear when you play the note in combination, which is what we do most of the time as tuners.

No need to get upset or dismayed. I mean, Physics gives us really weird stuff, like Relativity, the Space-Time Continuum etc. The concept of the First partial of a string being sharper or flatter than the Average sharpness/flatness of the strings part(ial)s, isn't really beyond comprehension. Just new.

Tuning text-books don't go into this. In fact the people writing about tuning probably haven't come across it. The sort of people in the tuning world who do know about it are the people who have developed Electronic Tuning Devices. They don't try to explain the concepts because their purpose is to get you started on using the device. And understanding isn't needed if you're intending to let an ETD take the place of your own brain.

Inharmonicity

In the time of the harpsichord and early pianos, strings were very light. A series of octaves – octave, double octave, triple octave could be tuned such that any pair of the four strings would be in tune.

As piano manufacturers started producing the louder, more stable instruments by fitting heavier gauge strings, the inconsistency within a series of octaves was noticed.

In the mid 19th Century, a French writer on tuning, declared that "modern manufacturers have made more augmented semitones". This was understood by a rival French writer on tuning [4] as "It is as if he were saying that one could alter the boundaries of the octave which contain the twelve semitones".

The phenomenon - **inharmonicity** was measurable after the development of the Stroboconn - the earliest Electronic Tuning Device, and research commissioned by the American Piano Wire Company in the 1940s established that the stiffness of a tuned string could be quantified for that string based on its length and diameter, and the sharpness of any partial could be modelled from this constant in a relationship which increases exponentially with the partial number.

[4] Claude Montal, The Art of Tuning one's own Piano, p.11

This research reached the piano tuning community in the late seventies in a series of articles in the American Piano Technician's Journal titled "The Calculating Technician", the main purpose of which was to assist technicians in redesigning pianos. By changing diameters and possibly lengths, the values for tension, breaking-strain, relative loudness and inharmonicity can be adjusted. The equations for Tension, and Stiffness given were as

$$T = (\frac{2^{\frac{m}{12}}Ld}{769})^2 \qquad \text{and} \qquad S = \frac{1731}{0.000198}.\frac{d^4}{L^2T}$$

where *m* is the note number as read from the keystick. The constant 769 in the first equation puts the value in millimetres and kilograms. The number *1731* in the Stiffness equation is *1200/log2* which puts the result into octave cents.

From *S* the Inharmonicity [5] for any partial *k* can be approximated: $I_k = S(k^2-1)$.

The frequencies of the partials within a stiff piano-string then, are sharper than simple multiples of the fundamental frequency.

So if a string has Stiffness of 0.1¢, the 2nd partial will be 0.3¢ sharp, the 3rd partial will be 0.8¢ sharp, and the fourth partial will be 1.5¢ sharp.

The equation for **I** is useful for rescaling a piano, and can also be used in a spreadsheet to calculate to some degree of reliable approximation the inharmonicity of the second and third partial. 'Paper' tunings can be calculated from the results[6].

The inharmonicity of some partials is also affected by the hardness of the hammers, the amount of the hammer surface which impacts the string, the point on the strings where the hammer strikes, and the elasticity of the strings, as well as the firmness of the contact of the string on the bridge.

The Elusive Octave

Every musician can tell you that the octave of a note vibrates at twice its frequency, and since the time of Pythagoras it has been known that halving the speaking length of a string (or monochord) will result in the octave.

Less known is the fact that the *sound* of an in-tune octave on the piano depends on the co-incidence of the 2nd partial of the base note with the 1st partial of its

[5] The approximation is *predictive*, and it takes the length, diameter and pitch as input. The results of real-time computer analysis use a different equation, with some similarities to this one. The inharmonicity coefficient – 0.000198 here – is tried and on all partials, and if there aren't enough matches for it, another value is tried, and another until the maximum number of partials can be matched.

[6] The earliest incarnation of the Reyburn CyberTuner used this method, and for some time was included with the laptop version of RCT as a curiosity.

octave, rather than the harmony of the frequency of the base note with twice that frequency in the octave above.

Even less obvious, is that the ‘in-tune’ sound of an octave also involves the co-incidence of the 4th partial of the base note with the 2nd partial of its octave, the 6th partial of the base note with the 3rd partial of its octave, the 8th partial with the 4th, the 12th partial with the 6th and so on.

If inharmonicity were not present, the coincidence of one set of partials would imply that any other pair of partials on the two strings would coincide. Since the partials on of a stiff piano string are inharmonic, with apparently random deviations, the alignment of one set, say the 4th partial of the base note with the 2nd partial of its octave, does not imply that another pair, for instance the 6th partial of the base note and the 3rd partial of the upper note, will coincide exactly. Fortunately our ears generally cope with the inconsistencies. However tuners must practice some discipline in the choice of partials to keep the tonality and pitch trend consistent.

From classical times to the late 19th century, the octave chord was thought to be fixed, having its upper note at half the length of the base note. In the mid-nineteenth century we have both Earl Stanhope and Claude Montal declaring that tuning the octave was the same as tuning a unison.

Totally unforeseen before the highly strung instruments of the late 19th century and onwards, is the fact that an octave can be *stretched* to different degrees, depending on which pairs of partials are aligned between them. It is up to the tuner to apply the checking techniques uniformly, to produce a consistently smooth tuning curve.

If an aural tuner uses only octave tuning without any checking chords, and the results are plotted on a graph, the tuning curve will show a saw-tooth pattern, suggesting among other faults, an inconsistent alignment of partials, and a varying tonality in the octave and other chords.

The partial checks mentioned here are now quite widely used, (though some schools have no systematic method for the highest and lowest octaves). The 3rd, 10th and 17th checking chords are either too fast or too slow to be useful in the extreme octaves. One tuning course on offer today advises its students that they are ‘on their own’ in the top octave, and in some parts of the world no-one expects the top octave to sound consistent.

Thankfully the designers of the Yamaha method discovered a set of checking chords for the treble, used in parallel with the 10th and 17ths until they become too fast to hear, involving the double octave, triple octave, and twelfth partials, and the tuning can be continued to the top, with reliable precision. No more “You’re on your own here, good luck!”

In the low bass, the tenths are audible, but very slow, increasing the time taken to assess them. Here the creators of the method added an interval not considered before, the seventh, separated by one or two octaves so that the upper note is in the temperament area. The speed of a seventh or 'extended seventh' is quite countable, and the lowest notes can be pitched neatly, allowing for consistent tonality of any interval, not just the octave.

Beyond the Temperament

Early in the 20th century it was noticed that if you tune F3–F4 as a simple 2:1 octave, and then tune F4 to F5 the same way, it is quite likely that the surrounding 4:1 double-octave, F3–F5, will sound flat. The solution was found to be in stretching the octaves slightly. This involves a small increase in the speed of the tenths and seventeenths compared to the speed of the thirds.

In comparing the speeds of C♯2 with F3 - F4 - F5 as an example you will have aligned the partials of the F's like this:

C♯2 partial 5	:	F3 partial 4
		F4 partial 2
		F5 partial 1

and you can describe the octave relationships in terms of the partials:

F3-F4 is a 4:2 octave
F4-F5 is a 2:1 octave, and
F3-F5 is a 4:1 double octave.

Aural Tuning Meter

In all likelihood, several people thought of this way of tuning at around the same time. The system of checking chords using the third-tenth-seventeenth evolved into a tuning method in which the notes are set by comparing the previous tenth or seventeenth, and making the current one slightly faster.

The same procedure – 'running' through a series of thirds, tenths or seventeenths is used as an aural pitch-measuring mechanism. This 'meter' works faster than any ETD. The results are obviously not in numbers, but are relative: "This note is flat/correct/sharp in relation to the previous note".

In our tuning school, the examiners first assessed a tuning starting with the third F3–A3 and running up the scale, swapping to tenths at A4, so instead of the third F4–A4, F3–A4 was used. Similarly at A5, the seventeenth F3–A5 was used, and the seventeenth continued to the top. The bass is assessed in the same way, starting with F3–A4, the span of the full temperament, and progressing downward.

Double versus Triple Octaves

F3-F5 is the first double-octave encountered after tuning the temperament. F7 is tuned off F5, and these three F's form a series. F6 is tuned from F4. so in effect we have two series of double-octaves between the temperament area and the top:

F3 - F5 - F7, and
F4 - F6.

At some point someone decided to check the triple octave F3-F6 when they got to F6, and realised that there is a slight beat there. As you move up from there, the slight beat gets faster, until if you compare C4 with C7, you can hear around 5 beats per second, even though the double octave C5-C7 sounds pure.

By F7 you might hear 10 or more beats, depending on the piano, and by C8 there could be up to 30 beats per second across three octaves, even though the double-octave is tuned pure.

So a series of notes tuned as double-octaves, when tested as triple-octaves show a beat, which is quite fast in the highest notes. There is a parallel in the triple-octave tuning method. As an example, C8 tuned as a triple octave will beat when tested as a double-octave, but less than it would in the other style, the reason being that in triple-octave tuning, the alto area ends up slightly sharper.

The double-octave style of tuning was the first 'stretched' tuning style to be developed after the simple octave style, and is probably the most common aural tuning style around.

Chapter 2

Triple Octave Tuning

The beats that you hear with the same note three octaves below 'progress' in the same way that the third and tenths 'run'. Meticulous double-octave tuners play the double octaves in a run, to make sure that they are beatless, then run through the triple octaves to make sure that the beats speed up.

I have no idea who first thought of swapping the roles of the double and triple octaves [1].

I do know that Yamaha introduced this style of tuning (as an option) in the Conservatorium Tuning School late in 1981. A year later the Tuning Graph was emended allowing students make a choice. Triple-octave tuning is known in the US, but has not yet caught on in that very conservative environment. Provision is made for the method in the various tuning apps, but it is not implemented very well.

Advantages of Triple Octave Tuning

There are several advantages to be had from Triple Octave Tuning: controlled octave stretching, efficient pitch-raising, cleaner unisons, and brighter tone.

For a start, the triple octave interval is slightly wider than the three separate octaves it contains. It is usually wider than the combined width of the double octave plus octave it contains, and also the octave plus double octave it contains.

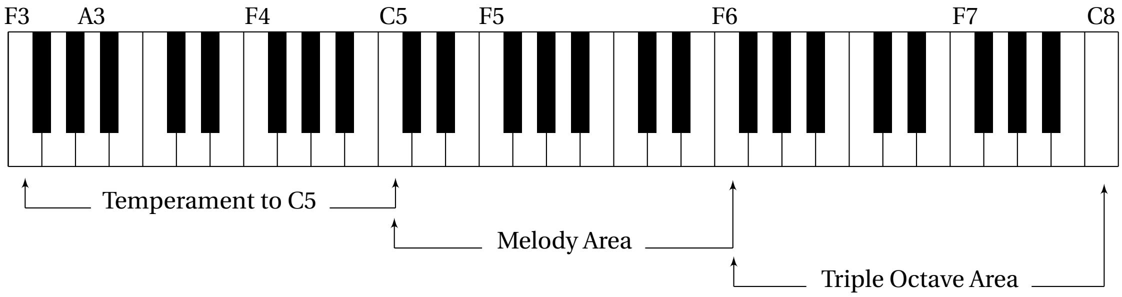
[1] In essence the idea is to check that a note, starting with F6, is exactly in tune ('pure') with its namesake 3 octaves below, but slightly sharp of its namesake 2 octaves down. As the tuning progresses towards the top, the *triple-octaves* stay beatless, while a distinct beat is audible with the double-octave below.

Take as an example the task of tuning the highest A (A7). If you tune it as a double octave from A5, then you are basing the tuning on a note which is removed from the A4 of the Temperament Area by an octave. On the other hand, if you use the Triple Octave method, then you tune A7 from A4, the top note of the Temperament Area. There are less intermediate steps, less chances for inaccuracy.

Even if your tuning decisions are all correct on your way up to the top, if the piano is quite flat, the middle part of the right hand is unstable. Referring the notes in the top octave to notes 3 octaves below bypasses this problem area. On the second pass, the top octave may not need any adjustment.

The dampers usually stop at D♯6 or E6. This means that the top 20 or so notes from F6 to C8 are free to vibrate in sympathy with their namesakes below. If this area is tuned to the a similar area 3 octaves below, they will be able to vibrate in sympathy with notes around middle C, so this area will perhaps have greater tonal enhancement.

Tuning to a note 3 octaves below is actually easier. If you're tuning C8 to C5, C5 has a very healthy spectrum of partials, and the matching of the appropriate partial of C5 to C8 is easier to recognise than matching C6 to C8 as in the double octave method. The amplitude of the partials in the Temperament Area is greater than the amplitude in the 'Melody Area'.



Clean Unisons

One advantage of Triple Octave tuning deserves a section of its own. Have you noticed how "noisy" some notes are in the high treble? A single string seems to clash with notes below, producing something a bit like false beats. The situation gets worse when you tune the other two strings to the first – there are beats all over the place.

The "clean unison" effect with triple-octave tuning is quite dramatic. If you have set the note nicely to its namesake three octaves below, it's easy to understand that it will sound clean and healthy when the triple octave interval is played.

It's harder to explain why the note sounds cleaner by itself when the lower note isn't being played at the same time.

Possibly the lower note still has an effect on the note three octaves higher, even though it is muted by the damper. Electronic measurements seem to suggest this may be the case.

Brighter Tone

This feature is related to the last. In essence, with the Triple-Octave method, we tune the *un-damped* "Top Twenty" notes from F6 to C8 to match the 8th partial of the same notes in the stable Temperament Area, F3 to C5. The Top Twenty sing in sympathy with these lower notes, adding to the total tonal strength of the piano.

With the Double-Octave method, the same Top Twenty vibrate sympathetically, but with notes an octave higher. The difference is minimal perhaps, but significant.

Definitions

We need a couple of definitions, to make it easier to talk, write or read about things. These terms were used in the Yamaha course and they make good sense:

- The Temperament as such is from F3 to F4. Fairly obvious.
- The Full Temperament. Extending the Temperament up to A4, gives us the Extended Temperament, or better the *Full* Temperament. [2]
- Temperament Area. We also used the term 'Temperament Area' to include three more notes above A4, to C5. So in tuning the highest notes of the piano, we would talk about *referring* those notes to the 'Temperament Area'.
  In general the idea is that these notes have a kind of authority, as being reliably in tune and unlikely to wander during the tuning.
- The Triple-Octave Area (a.k.a the Top Twenty) starts at F6 and goes to the top.
- The Double-Octave Area. Below the Triple-Octave Area we have the Double-Octave Area, starting with F5, and going up to F6.
- The Octave Area. Between A4 and F5 we have another area, consisting of 8 notes which are tuned as *slightly wide* octaves, and checked as fourths,

[2] These were the terms used in conversation in the Course. If I were talking to someone from outside this tradition, I might say 'Temperament, *including* A4.

> fifths and tenths. In Double-Octave tuning, this area is tuned to be precisely in tune with the octave below, but also with reference to the tenth interval, which has priority over the octave. In Triple-Octave tuning, the Fifth, or Twelfth[3] takes the priority over the Octave, making this area *theoretically* 2¢ sharper than it might have been.

This area which is slightly sharper makes a bridge to the Triple-Octave area which begins at F6.

Method

The Temperament, F3 – F4

I'm assuming you have this well under control. I have a chapter on it, which I'm in the process of revising. I can add this in later if you like.

Extended Temperament, F♯4 – A4

The inharmonicity is greater here than in the lower part of the temperament area, so set speeds are irrelevant. Instead you have to rely on hearing a progression of speeds in the sixths, while keeping the pitch of a note not sharper than pure as a fifth.

The sixths are used instead of thirds, because their speeds are slower, making it easier to assess the progession, or *acceleration*.

Steps 14, 15 and 16:

Tune: F♯4, G4, G♯4 as fifths and as fourths from the temperament. If you hear 1 beat with the fourth, and *no* beats on the fifth, then the sixths should progress smoothly.

Test: Optionally test as octaves. With practice you can skip this.

Compare: Carefully check the progression of speed in the sixths, starting with G♯3–F4 up to B3–G♯4.

---

[3] Octave + Fifth

The beat-speed of the fourths increases, in theory, in the temperament. It would be reasonable to assume that the fourths and fifths increase in speed in this area as well.

In practice, because of the increase in Inharmonicity here, the routine here is to pull the note up as a fifth, without any beats. The pure sounding fifth has quite a wide range, and it's quite easy to tune it too sharp. So we check the note as a fourth, and make sure that it isn't beating too fast. Generally 1 to 2 beats a second on all these notes is a good first guess. The main decision can be made by checking that the note sounds right as a sixth, slightly sharper than the sixth below.

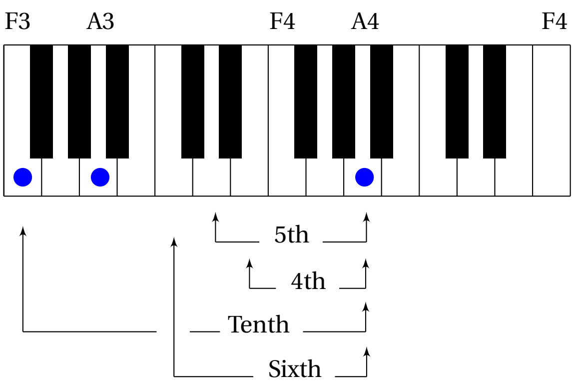
Reminder 1. Pull the note up as a fifth. 2. Check it's not too sharp, as a fourth. Check the speed as a sixth.

Step 17:

Tune: A4 as a fifth from D4 and as a fourth from E4

Test: The third F3–A3 with the tenth F3–A4 should have the same speed.

By aligning the speed of the third and the tenth in this way, you establish a 4:2 octave between A3 and A4. Having determined that important interval, it is then possible to reassess the sixths A3–F♯4, A♯3–G4 and B3–G♯4, making sure that they accelerate to fit in with C4–A4.



The 5th and 4th are used to tune the note, while the 10th and 6th are used to check it, possibly before tuning, and certainly after tuning it.

If the piano has been recently tuned, some sections may still be in tune, so you might briefly check the tuning with the Tenth. If you know that the notes are going to be out, skip this and start with the tuning intervals – the fourth and the fifth – to save time. In general, when the fifth is beatless, the fourth will beat at a rate of 1 or 2 beats per second. If the section is particularly flat, it may help to give the fourth 3 beats per second, in anticipation of a drop in pitch. This can be checked and fixed on the second pass.

Steps 18 and 19:

Tune: A♯4 and B4 in the same way as A4
Test: The sixths and tenths should 'run'. However. it is not necessary to keep the 10ths at the same speed as the third, as with A4.

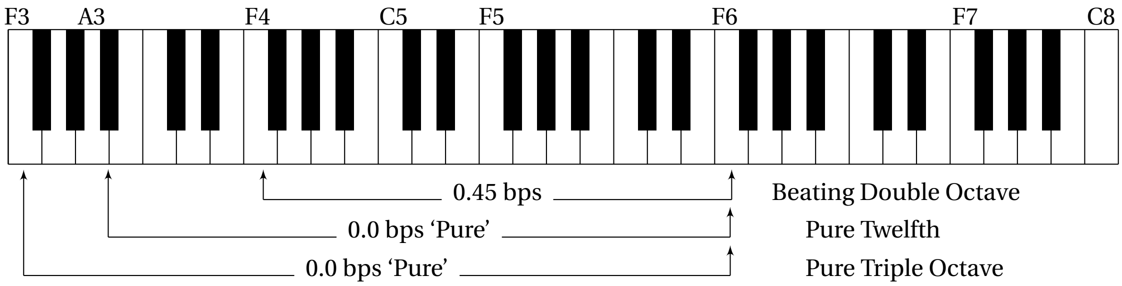
You could consider B to be A♯♯– the method is the same for all three A4's. As with A4, the 5th and 4th are used to tune the note, while the 10th and 6th are used to check it, possibly before tuning, and certainly after tuning it.

Because the Triple Octave tuning curve is slightly sharper than with a Double Octave tuning, we introduce some sharpness here after A4. A4 needs to be kept beatless with the octave below for practical reasons. For a start the pitch of the starting note A3 is set from A4, which in turn is set from some external source

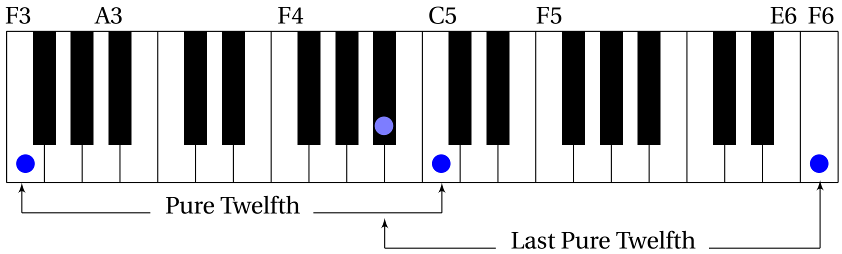
NOTE—. These days many musicians carry tuning devices on their phones, and our work is liable to be judged by the pitch of A4. The inharmonicity of the partials may suggest a slightly sharper tuning, but this can't be foreseen at the outset, so it is a good idea to discipline yourself to have A4 tuned precisely. However if the first pass involves a pitch raise of more than a couple of cycles, it can be helpful to sharpen A4 in accordance with the requirements of the partials' inharmonicity, to note the sharpness in relation to A3, and to reproduce this in the tuning of A3 to A4 the second time around. You can also tune A3 from A4 as an 8:4 octave, which is slightly wider than 4:2, at the outset. The 8:4 octave is described in Section 'whatever'.

Strategy for tuning C5 to F6

When we get to F6, it can be seen that a note tuned pure with its namesake 3 octaves below beats with its namesake 2 octaves below, but sounds pure with the note a twelfth below.



When tuning the section from C5 to E6, we have no triple octave namesake to tune from, so we use the 'beatless' twelfth in this section.



Tuning a Pure Twelfth

Version 1. Partials 6:2

A tempered 5th – or 12th – beats quite slowly. Assessing a perfect 5th can take quite a while, as the beat gets slower and slower until finally there is no beat at all. Precious time is wasted, considering that the same process has to be applied to several notes.

A fast, accurate check is available, in the form of the *Equal Beating* minor triad, widened so that the top note is an octave higher than in the basic triad.

A minor triad based on F3 would be F3–G♯–C4 [4]. You will recall that a minor triad contains its signature minor third based on its tonic, and a major third with the fifth note as its top member. If the triad is widened, the top note becomes C5, and forms a tenth.

The partials involved, and their Theoretical [5] Equal Temperament frequencies are:

F3 :6 = 1047.655
G♯3 :5 = 1038.262
C5 :2 = 1046.502

The beats heard between the minor third part of the triad, F3 and G♯3 are:

1047.65 - 1038.26 = 9.39 beats per second,

while the beats in the major third/tenth part are:

1046.50 - 1038.26 = 8.24 beats per second.

The difference is:

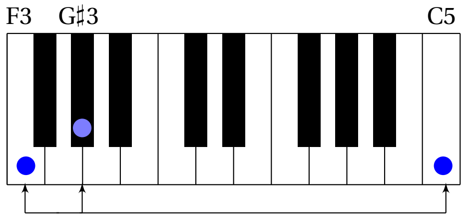
9.39 - 8.24 = 1.15 beats per second.

[4] Of course in Harmony, the G♯ should be called A♭ here. However, tuning discussion dispenses with the flats – the black keys are ‘sharps’ after all.

[5] At A440 and without Inharmonicity. In the Inharmony App, “Theoretical Equal Temperament” is abbreviated to “ET”.

So the minor third and the major third contained within a minor triad differ by a little over 1 beat per second, at the level of F3. This represents a slight clash, but passes unnoticed. If the top note of the triad in the wide or extended version is sharpened such that both thirds beat equally, the triad actually sounds slightly more pleasant.

In summary you are tuning the notes C5 to E6 with a combination of pure (or almost pure) fifths, checked from excess as fourths, and double checked as tenths. The speed of the tenths, instead of being left to intuition, can be verified by comparing the speed of the tenth with the speed of the minor third interval below the tenth.

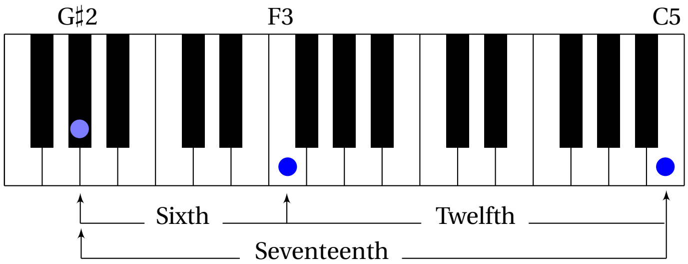


Equal-beating minor third and tenth, producing a pure 6:2 Twelfth

The checking technique is to play the minor third, which has already been set, so it can be considered to be correct for the particular piano, and count 3 or 4 beats, and then play the tenth counting the same number of beats and judging with your sense of musical time, whether the same amount of time has passed.You can alternate between the minor third and the tenth a number of times, and see if the alternating chords are in time.

Version 2. Partials 3:1

There's a variation of this partial check, which can be used when the beats get too fast to count.



Equal-beating major sixth and seventeenth, producing a pure 3:1 Twelfth

The same notes are involved, but the minor third note – G♯ in our example, is taken down an octave. G♯ may not have been tuned if this is the first pass, but the speed can be used in the check, as long is it is reasonably close . At this level, it makes F3 into a sixth, and C5 becomes a seventeenth.

This arrangement involves a different set of partials. The partials of the twelfth are now 3:1, and as these are of a lower order compared to the 6:2 partials in Version 1, the inharmonicity is not as great, so the spread in pitch of the two notes is not as great. Both forms of twelfth (or a fifth tuned along the same lines) sound pure. It is similar to the situation with octaves: you can have an octave[6] tuned by aligning partials 2:1, 4:2, 6:3 using different checking techniques.

In most registers you can't tell the difference, nor can you detect a clash, as the congruence of the partials you choose to match masks the slight beat between the partial you chose to ignore. The sixth-seventeenth check beats more slowly, so it's useful higher up.

> NOTE—. In the Yamaha Way of tuning, with its emphasis on speed, these checks aren't used. The octave, twelfth and tenth (or seventeenth) are thought to be enough. The tuner learns to feel the speeds of the tenths by comparing each tenth with the previous one. You are encouraged to memorize the speed of the current tenth and mentally compare it when you move on to the next note. All this is practiced over hundreds of supervised tunings.
>
> However we haven't got the luxury of going back to the beginning like that. Comparing the sixth with the seventeenth doesn't take long, but the process will add a couple of minutes to a tuning. A small price for greater accuracy.
>
> Besides, inaccuracy can creep in. If you do need to hurry, perhaps during the 'first pass' tuning, use these checks every 2 or 3 notes, and rely on the accelerating tenth in between.

[6] I need to do a section on this topic

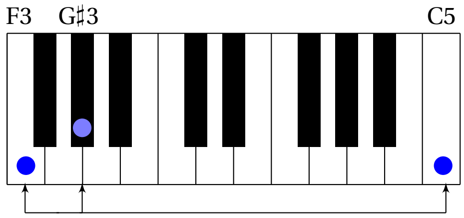
Procedure for tuning from C5 to F5

Step 21:

Tune: C5 as a 'wide' octave, then check that it 'runs' as a tenth.

Test 1: Test that the tenth with G♯3 beats at the same speed as the minor third F3–G♯3.

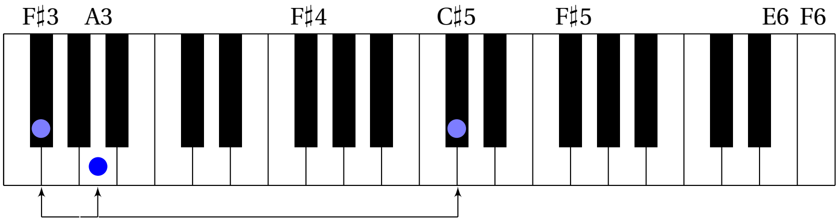
NOTE—. On some pianos, especially when *voicing* is needed, the pure twelfth can be a bit too sharp. It's a good idea to check the sound of the fourth with G4 to see that it isn't beating excessively fast.



Step 22:

Tune: C♯5 as a 'wide' octave, then check that it 'runs' as a tenth.

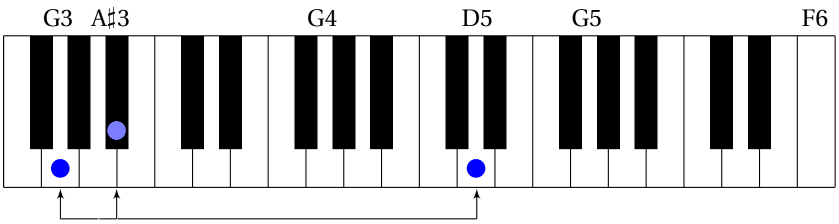
Test 1: Test that the tenth with A3 beats no faster than the minor third F♯3–A3.



Step 23:

Tune: D5 as a 'wide' octave, then check that it 'runs' as a tenth.

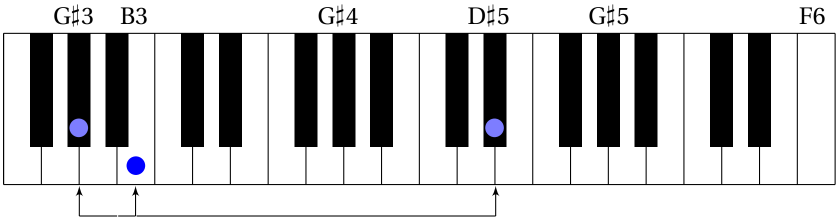
Test 1: Test that the tenth with A♯3 beats no faster than the minor third G3–A♯3.



Step 24:

Tune: D♯5 as a 'wide' octave, then check that it 'runs' as a tenth.

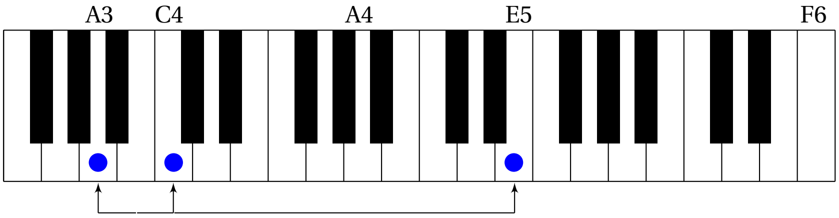
Test 1: Test that the tenth with B3 beats no faster than the minor third G♯3–B3.



Step 25:

Tune: E5 as a 'wide' octave, then check that it 'runs' as a tenth.

Test 1: Test that the tenth with C4 beats no faster than the minor third A3–C4.



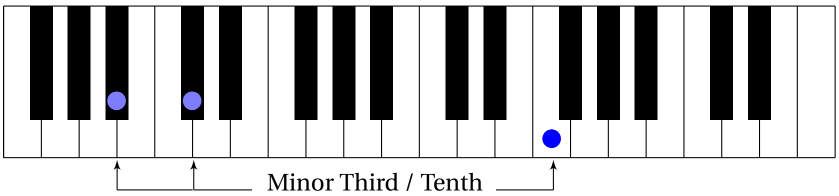
Procedure for tuning from F5 to F6

At F5 another check becomes available – the double-octave with F3, the lowest note tuned so far. In a conservative double-octave style tuning, the double-octave interval is tuned pure. In the triple-octave 'Concert' style, the twelfth is tuned pure, so F5 is tuned beatless with A♯3. In the Temperament A♯3 was tuned with a slow beat, slightly sharp of F3. The same beat can be heard in the double-octave interval F3–F5.

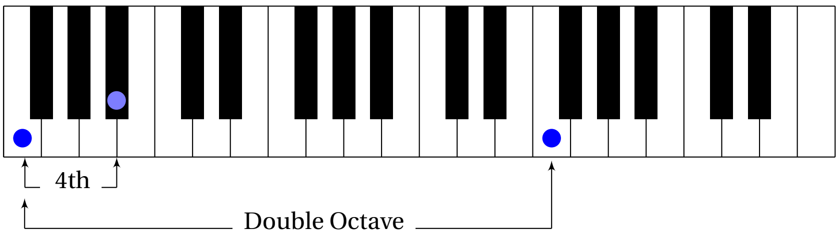
Step 26:

Tune: F5 as a 'wide' octave, then check that it 'runs' as a tenth.

Test 1: The tenth with C♯4 should beat no faster than the minor third A♯3–C♯4.



Test 2: The double-octave F3–F5 should beat at the same speed as the fourth F3–A♯3.



Order: In the Yamaha Way, the following intervals are played in strict order, and make a kind of melody. By keeping the method consistent, you learn to recognise the beats more easily. Muscle memory also plays a part, helping you to acquire speed.

1.	Previous Tenth	C4–E5	Play
2.	This Tenth	C♯4–F5	Compare
3.	Octave	F4–F5	Set
4.	Tenth	C♯4–F5	Check
5.	Twelfth	A♯4–F5	Pure
6.	Double Octave	F3–F5	Beating

Comments:

Play the previously set tenth, C4–E5, then C♯4–F5 to get an idea of how sharp or flat F5 is. It may even be right already.

Set F5 from F4 (or A♯4). You can use the 5th instead of the octave to set a note.

Check the tenth again. You can compare it with the minor third as in the keyboard diagram. This check isn't part of the *Yamaha Way*, but is a very handy tool.

Next the twelfth with A♯3. Note whether the twelfth is pure, and if so, play the Double-Octave with F3 and listen to the beat. It should be around 1 beat in 2 seconds. You can remind yourself of the beat required by playing the F3–A♯3 fourth from the Temperament.

Finally play the tenth C♯4–F5 and move on to the next tenth.

More accurate variant: The same order is used, but when setting the Tenth, it can be compared with the minor third, as in the keyboard diagram. This might add a minute or two to the tuning, but the pay-off is greater accuracy. In the next steps, I'll use this alternative order:

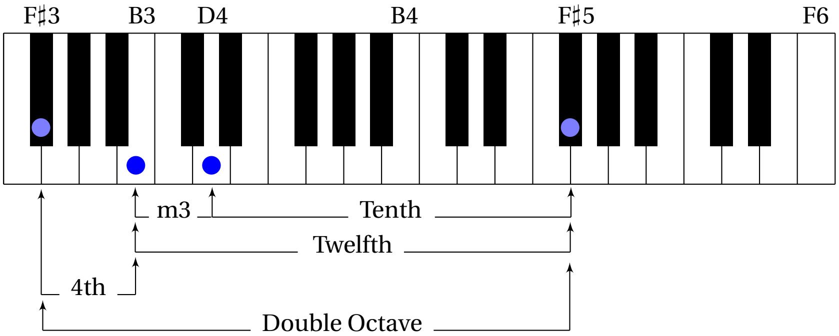
1.	Previous Tenth	C4–E5	Compare
2.	This Tenth	C♯4–F5	Compare
3.	Octave	F4–F5	Set
4.	Tenth	C♯4–F5	Compare with minor third A♯3–C♯4
5.	Twelfth	A♯4–F5	Check
6.	Double Octave	F3–F5	Check
7.	Tenth	C♯4–F5	Check

Step 27:

Tune: F♯5, then check that it 'runs' as a tenth and beats as a double-octave.

Test 1: The tenth with D4 should beat no faster than the minor third B3–D4.

Test 2: The double-octave F♯3–F♯5 should beat at the same speed as the fourth F♯3–B3.



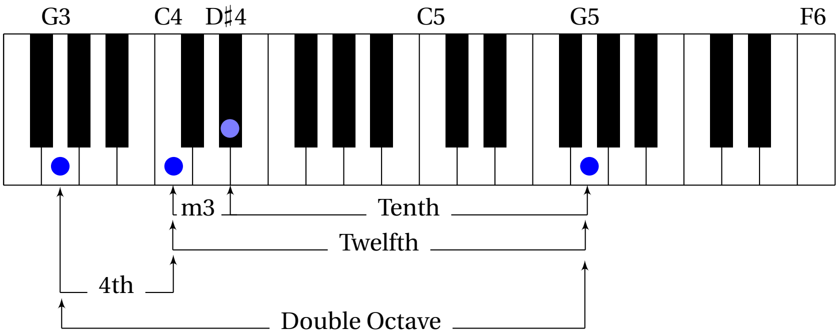
Order: Play the intervals in strict order, keeping the method consistent.

2.	This Tenth	D4–F♯5	Compare C♯4–F5
3.	Octave	F♯4–F♯5	Set
4.	Tenth	D4–F♯5	Compare
(4a	Minor third	B3–D4	Compare)
5.	Twelfth	B4–F♯5	Pure
6.	Double Octave	F♯3–F♯5	Beating

Step 28:

Tune: G5, then check that it 'runs' as a tenth and beats as a double-octave.

Test 1: The tenth with D♯4 should beat no faster than the minor third C4–D♯4.
Test 2: The double-octave G3–G5 should beat the same as the fourth G3–C4.
Test 3: The twelfth with C4 should be pure.



Order: Play the intervals in strict order, keeping the method consistent.

1.	Previous Tenth	D4–F♯5	
2.	This Tenth	D♯4–G5	Compare
3.	Octave	G4–G5	Set
4.	Tenth	D♯4–G5	Check
(4a	Minor third	C4–D♯4	Compare)
5.	Twelfth	C4–G5	Pure
6.	Double Octave	G3–G5	Beating

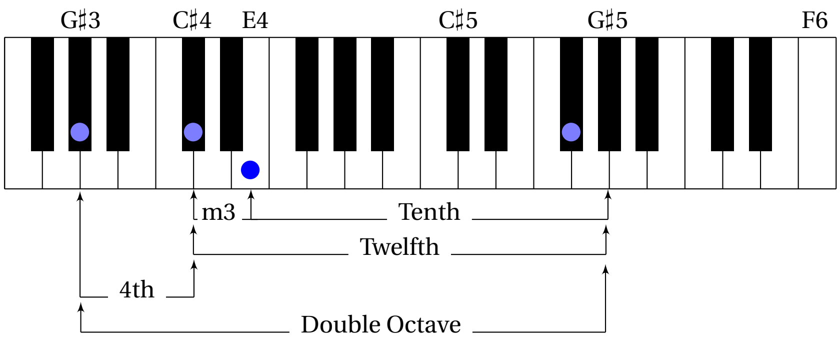
Step 29:

Tune: G♯5, then check that it 'runs' as a tenth and beats as a double-octave.

Test 1: The tenth with E4 should beat no faster than the minor third C♯4–E4.

Test 2: The double-octave G♯3–5 should beat the same as the fourth G♯3–C♯4.

Test 3: The twelfth with C♯4 should be pure.



Order: Play the intervals in strict order, keeping the method consistent.

1.	Previous Tenth	D♯4–G5	
2.	This Tenth	E4–G♯5	Compare
3.	Octave	G♯4–G♯5	Set
4.	Tenth	E4–G♯5	Check
(4a	Minor third	C♯4–E4	Compare)
5.	Twelfth	C♯4–G♯5	Pure
6.	Double Octave	G♯3–G♯5	Beating

Hand-over

With A5 a new interval check is available, the seventeenth F3–A5. However, we have been using the tenth up till now, so with A5 both checks are used. The tenth is used first, that is the what has been in use up to now, and the usual comparison with the previous tenth is made.

A5 is checked with both the tenth and the seventeenth, but from A♯5 on the seventeenth only is used. You will notice it is slower and easier to assess, and it has more *authority* as the lower note is from the temperament.

An additional speed-check is available: After playing the seventeenth with F3, the twelfth can be checked by playing the sixth F3–D4. The seventeenth and the sixth should beat at the same speed. You may have to compromise this with poorly designed pianos, or if the hammers badly need voicing.

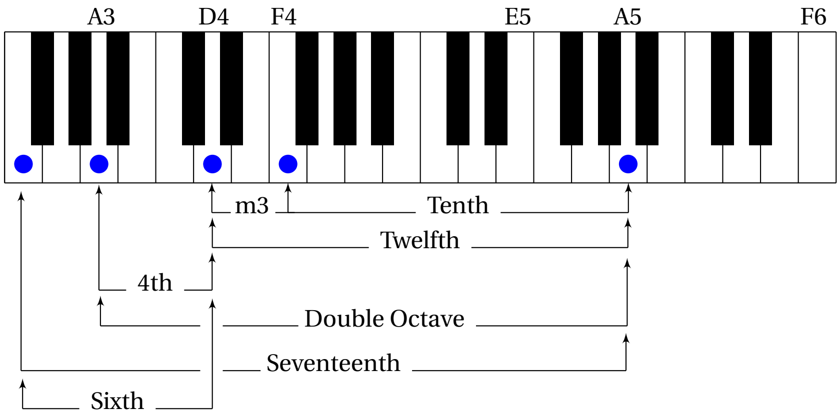
Step 30:

Tune: A5, then check that it 'runs' as a tenth and beats as a double-octave.

Test 1: The double-octave A3–A5 should beat the same as the fourth A3–D4.

Test 2: The twelfth with D4 should be pure.

Assess: The seventeenth with F3. It should beat the same as the sixth F3–D4.



Order: Play the checks in the order given. Consistency builds speed.

1.	Previous Tenth	E4–G♯5	
2.	This Tenth	F4–A5	Compare
3.	Octave	A4–A5	Set
4.	Double Octave	A3–A5	Beating
5.	Twelfth	D4–A5	Pure
6.	Seventeenth	F3–A5	Assess
7.	Sixth	F3–D4	Compare

Note The seventeenth F3–A5 will probably beat faster than the tenth F3–A4, even though in theory the speed is the same.

After A5 the procedure dispenses with the tenth check. So